Lectures on Dynamic Contracts and Mechanism Design

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High level motivation

The one unifying idea underlying this primer is that of *optimal economic distortions*. For an object (or allocation) valued at θ by an agent, Myersonian mechanism design (Myerson [1981]) defines the concept of virtual valuation as:

$$v(\theta) = v^e(\theta) - d(\theta)$$

where $v^e(\theta)$ is the efficient allocation and $d(\theta)$ is the distortion derived as part of the constrained optimization problem. The (optimal) distortion is the mirror image of the information rent given to the agent, the main source of friction in these models.

If we were in a world of perfect information, then maximal economic surplus can be generated, $d(\theta) = 0 \;\forall\, \theta$. Moreover, if the bargaining power between the principal and agent(s) is "fair", it would lead to the sustenance of institutions that would split the surplus reasonably. Alas that is rarely the case in the sense that asymmetric information is a widely prevalent phenomenon is most economic transactions (think of buying a house or paying an employee), and bargaining powers are rarely reasonable. In fact the role of institutions (such as taxation and regulation) is to work to minimize the asymmetric information problem and ensure fair splitting of the economic pie. Remember, the beautiful Myerson and Satterthwaite [1983] result tells you that under some reasonable institutional assumptions, efficiency, i.e. $d(\theta) = 0 \;\forall\, \theta$, cannot be attained.

These realities are often captured in mechanism design through the concept of the virtual value. For instance, in an auction, the seller may not assign the object to the person with the highest value for she does not know the bidders' actual value, but one with the highest virtual value which in turn informs how the bidders bid in the auction. Seller may even decide to withhold the object by

*I'm indebted to my co-authors Marco Battaglini and Ilia Krasikov for teaching me much about dynamic contracts and mechanism design

having a reserve price, which is higher than her opportunity cost of holding it. As a consequence these assignment are inefficient, economic surplus is lost, but it is the best a revenue maximizing seller can do.

Much of dynamic mechanism design is concerned with finding the corresponding dynamic virtual value, which typically takes the following form:

$$v(\theta|h^t) = v^e(\theta|h^t) - d(\theta|h^t)$$

where as in the static problem, $v^e(\theta|h^t)$ is the efficient allocation and $d(\theta|h^t)$ is the distortion derived as part of the constrained optimization problem. In most examples we will look at the concept of efficiency will be static: $v^e(\theta|h^t) = v^e(\theta)$, and the challenge will either be to calculate the optimal dynamic distortions: $d(\theta|h^t)$ or find out when a specific dynamic distortion, such as the efficient one, $d(\theta|h^t) \equiv 0$, can be implemented.

Seven key ingredients of the model will interact to produce different optimal values of d or different conditions on primitives for achieving a fixed value of what value of d:

- *Preferences*: as in the standard static model, these will typically be quasi-linear and symmetric rates of discounting.

- *Time horizon*: the design problem must have a future component, either agents information changes, or they arrive randomly, or the time of sale is in the future, etc.

- *Information*: how does the agent's private information evolve over time; it will be assumed to be drawn from a prior and then evolve according to an exogenous Markov process.

- *Incentives*: which concept of incentive compatibility is used; it will typically be the one corresponding to perfect Bayesian equilibrium and sometime we will also invoke to a stronger version of ex post equilibrium.

- *Feasibility*: what institutional constraints are imposed on the mechanism: such as individual rationality, budget balance, and promise keeping. Since this is dynamic model, feasibility constraints can be written down in many ways: for example do we impose ex ante, interim or ex post budget balance.

- *Objective*: this would typically be either maximizing the principal's profit, or implementing some notion of efficiency.

- *Commitment*: typically we will assume commitment on the side of the principal (to a contact or mechanism) and limited commitment on the side of the agent(s). This will help us get crisp solutions, which are useful, at least as benchmarks, sometimes more.

There are some excellent surveys out there on dynamic mechanism design: Bergemann and Said [2011], Vohra [2012], Krähmer and Strausz [2015a], Pavan [2016], and Bergemann and

Välimäki [2019]. If you seeing this stuff for the first time, I'd recommend starting first with two papers—Courty and Li [2000] and Battaglini [2005], and then reading through the most recent survey, Bergemann and Välimäki [2019].

In the list above, the two key ingredients that make the design problem "dynamic" are time and information. For example, Bergemann and Välimäki [2019]. categorize dynamic mechanism design thus:

> "In all of the above applications, the types of some agents and/or the set of allocations available change in a nontrivial manner across periods. For us, this is the distinguishing feature of dynamic mechanism design."

Lecture 1

1. Quick revision of static mechanism design: Laffont and Martimort [2002], Chapter 2, Börger [2015] Chapter 2.

2. Two-types two period and infinite horizon model: Courty and Li [2000], Battaglini [2005].

3. Dynamic envelope theorem and known examples of optimal contract solvable using the first-order approach: Besanko [1985], Boleslavsky and Said [2013], Pavan, Segal, and Toikka [2014], Battaglini and Lamba [2019].

4. Independent shocks approach: Esö and Szentes [2007], Esö and Szentes [2017].

Lecture 2

5. Dynamic payoff equivalence and implementing the efficient allocation in multi-agents dynamic mechanism design: Athey and Miller [2007], Athey and Segal [2013], Skrzypacz and Toikka [2015], Lamba [2022].

6. Recursive approach and financial constraints: Clementi and Hopenhayn [2006], Krähmer and Strausz [2015b], Krishna, Lopomo, and Taylor [2013], Krasikov and Lamba [2021], Krasikov, Lamba, and Mettral [2022].

Lecture 3

7. Simple economics of dynamic pricing and global incentive constraints: Deb [2014], Garrett, Pavan, and Toikka [2018], Krasikov and Lamba [2022], Li and Shi [2022].

8. Dynamic arrivals: Bergemann and Välimäki [2010], Pai and Vohra [2013], Board and Skrzypacz [2016], Garrett [2016], Kapon [2023].

9. Renegotiation, liquidation, relaxing commitment, and ratchet effect: Laffont and Tirole [1990a,b], Bester and Strausz [2001], Clementi and Hopenhayn [2006], Battaglini [2007], Maestri [2017], Doval and Skreta [2022].

Other interesting directions

10. Dynamic mechanisms without transfers: Li, Matouschek, and Powell [2017]. , Guo and Hörner [2018], Deb, Pai, and Said [2018], Lipnowski and Ramos [2020].

11. Experimentation and dynamic mechanisms: Guo [2016], Halac, Kartik, and Liu [2017], McClellan [2022].

12. Modeling in continuous time: Sannikov [2008], Williams [2011], Bergemann and Strack [2015], Krasikov and Lamba [2022], Bloedel, Krishna, and Strulovici [2023].

13. Dynamic financial contracting: Demarzo and Sannikov [2006], DeMarzo and Fishman [2007], Biais, Mariotti, Plantin, and Rochet [2007], Fu and Krishna [2019].

14. Robust dynamic mechanisms: Penta [2015], Mu and Libgober [2021], Chassang and Kapon [2022].

15. Information design and dynamic mechanisms: Li and Shi [2017], Wangenheim [2017], Ely and Szydlowski [2020],

16. Risk averse preferences: Thomas and Worrall [1990], Farhi and Werning [2007], Arve and Martimort [2016], Bloedel, Krishna, and Leukhina [2021], Luz [2022].

17. Applications to optimal taxation: Kocherlakota [2010], Farhi and Werning [2013], Golosov, Troshkin, and Tsyvinski [2016], Stantcheva [2018].

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